DIFERENCIÁLNÍ ROVNICE 1. ŘÁDU

1. Metodou separace proměnných řešte diferenciální rovnice:

a) $(x+1)y'-4y=0$

$$(x+1)\cdot\frac{dy}{dx}=4y$$

$$(x+1)\,dy=4y\,dx$$

$$\int\frac{dy}{y}=\int\frac{4}{x+1}\,dx$$

$$\ln|y|=4\ln|x+1|+c \qquad c=\ln K,\ K>0$$

$$\ln|y|=\ln|x+1|^4+\ln K$$

$$\ln|y|=\ln|x+1|^4\cdot K$$

$$|y|=|x+1|^4\cdot K \qquad K=H,\ H\in R$$

$$y=(x+1)^4\cdot H$$

$$\underline{\underline{y=H(x+1)^4}}$$

b) $3y'-\sin^3 x\cdot\cos x=0$

$$3\cdot\frac{dy}{dx}=\sin^3 x\cdot\cos x$$

$$3\,dy=\sin^3 x\cdot\cos x\cdot dx$$

$$\int dy=\int\frac{\sin^3 x\cdot\cos x}{3}\,dx$$

$$y=\frac{1}{3}\int\sin^3 x\cdot\cos x\,dx \qquad \left|\begin{matrix}t=\sin x\\ dt=\cos x\,dx\end{matrix}\right|$$

$$y=\frac{1}{3}\int t^3\cdot dt$$

$$y=\frac{1}{3}\cdot\frac{t^4}{4}+c$$

$$\underline{\underline{y=\frac{\sin^4 x}{12}+c}}$$

c) $xy' - 5\ln^4 x = 0$

$$x \cdot \frac{dy}{dx} = 5\ln^4 x$$

$$x \cdot dy = 5\ln^4 x \, dx$$

$$\int dy = \int \frac{5\ln^4 x}{x} dx$$

$$y = 5 \cdot \int \frac{\ln^4 x}{x} dx \quad \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x} dx \end{array} \right|$$

$$y = 5 \cdot \int t^4 \cdot dt$$

$$y = 5 \cdot \frac{t^5}{5} + c$$

$$\underline{y = \ln^5 x + c}$$

d) $2\sqrt{x} \cdot y' - \cos^2 y = 0$

$$2\sqrt{x} \cdot \frac{dy}{dx} = \cos^2 y$$

$$2\sqrt{x}\, dy = \cos^2 y \, dx$$

$$\int \frac{dy}{\cos^2 y} = \int \frac{dx}{2\sqrt{x}}$$

$$\operatorname{tg} y = \frac{1}{2} \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\operatorname{tg} y = \sqrt{x} + c$$

$$\underline{y = \operatorname{arctg}(\sqrt{x} + c)}$$

e) $e^y \cdot (x^2+1) \cdot y' - x^2 = 0$

$$e^y \cdot (x^2+1) \cdot \frac{dy}{dx} = x^2$$

$$\int e^y \cdot dy = \int \frac{x^2+1-1}{x^2+1} dx$$

$$e^y = \int \left(1 - \frac{1}{x^2+1}\right) dx$$

$$e^y = x - \operatorname{arctg} x + c \quad / \cdot \ln$$

$$\ln e^y = \ln |x - \operatorname{arctg} x + c|$$

$$y \cdot \ln e = \ln |x - \operatorname{arctg} x + c|$$

$$\underline{y = \ln |x - \operatorname{arctg} x + c|}$$

f) $(x\ln x+2x)\cdot y'=y$

$$(x\ln x+2x)\cdot\frac{dy}{dx}=y$$

$$\frac{dy}{y}=\frac{1}{x\ln x+2x}\,dx$$

$$\int\frac{dy}{y}=\int\frac{1}{x(\ln x+2)}\,dx \qquad \left|\begin{array}{l} t=\ln x+2 \\ dt=\frac{1}{x}\,dx \end{array}\right|$$

$$\ln|y|=\int\frac{1}{t}\cdot dt$$

$$\ln|y|=\ln|t|+c \qquad c=\ln k,\ k>0$$

$$\ln|y|=\ln|\ln x+2|+\ln k$$

$$\ln|y|=\ln|\ln x+2|\cdot K \qquad K=H,\ H\in\mathbb{R}$$

$$\underline{y=H\cdot|\ln x+2|}$$

g) $y'=x\cdot\sin x\cdot 2y$

$$\frac{dy}{dx}=x\cdot\sin x\cdot 2y$$

per partes

$$\int\frac{dy}{y}=\int 2x\sin x\,dx \qquad \left|\begin{array}{ll} u=2x & v'=\sin x \\ u'=2 & v=-\cos x \end{array}\right|$$

$$\ln|y|=2x\cdot(-\cos x)-\int 2\cdot(-\cos x)\,dx$$

$$\ln|y|=-2x\cos x+2\sin x+c$$

$$|y|=e^{-2x\cdot\cos x+2\sin x+c}$$

$$|y|=e^{2(\sin x-x\cos x)}\cdot e^{c} \qquad \pm e^{c}=K$$

$$\underline{\underline{y=K\cdot e^{2(\sin x-x\cos x)}}}$$

2. Metodou separace proměnných řešte diferenciální rovnice:

a) $y' + y \cdot tgx = 0$

$\frac{dy}{dx} = -y\,tgx$

$\int \frac{dy}{y} = \int -tg\,dx$

$\ln|y| = \ln|\cos x| + c \quad c = \ln K, K > 0$

$\ln|y| = \ln|\cos x| + \ln K$

$\ln|y| = \ln|\cos x| \cdot K \quad K = H, H \in R$

$\underline{y = H \cdot \cos x}$

b) $y' + 2y = 0$

$\frac{dy}{dx} = -2y$

$\int \frac{dy}{y} = \int -2dx$

$\ln|y| = -2x + c$

$|y| = e^{-2x+c} \quad \pm e^c = K$

$\underline{y = K \cdot e^{-2x}}$

c) $xy' + 2y = 0$

$x \cdot \frac{dy}{dx} = -2y$

$\int \frac{dy}{y} = \int \frac{2}{x} dx$

$\ln|y| = -2\ln|x| + c \quad c = \ln K, K > 0$

$\ln|y| = \ln|x|^{-2} + \ln K$

$\ln|y| = \ln|x|^{-2} \cdot K \quad K = H, H \in R$

$y = H \cdot x^{-2}$

$\underline{y = \frac{H}{x^2}}$

d) $y' - 3x^2y = 0$

$\frac{dy}{dx} = 3x^2y$

$\int \frac{dy}{y} = \int 3x^2 dx$

$\ln|y| = 3\frac{x^3}{3} + c$

$\ln|y| = x^3 + c$

$|y| = e^{x^3+c} \quad \pm e^c = K$

$\underline{y = K \cdot e^{x^3}}$

e) $y' \cdot \sin x - y \cdot \cos x = 0$

$\frac{dy}{dx} \cdot \sin x = y \cdot \cos x$

$\int \frac{dy}{y} = \int \frac{\cos x}{\sin x} dx$

$\ln|y| = \ln|\sin x| + c \quad c = \ln K, K > 0$

$\ln|y| = \ln|\sin x| + \ln K$

$\ln|y| = \ln|\sin x| \cdot K \quad K = H, H \in R$

$\underline{y = H \cdot \sin x}$

f) $y' - y \cdot \cos x = 0$

$\frac{dy}{dx} = y \cdot \cos x$

$\int \frac{dy}{y} = \int \cos x\, dx$

$\ln|y| = \sin x + c$

$|y| = e^{\sin x + c}$

$|y| = e^{\sin x} \cdot e^c \quad \pm e^c = K$

$y = e^{\sin x} \cdot K$

$\underline{y = K \cdot e^{\sin x}}$

g) $y' - \frac{y}{\sqrt{1-x^2}} = 0$

$$\frac{dy}{dx} = \frac{y}{\sqrt{1-x^2}}$$

$$\int \frac{dy}{y} = \int \frac{1}{\sqrt{1-x^2}}\, dx$$

$$\ln|y| = \arcsin x + c$$

$$|y| = e^{\arcsin x + c}$$

$$|y| = e^{\arcsin x} \cdot e^{c} \qquad \pm e^{c} = K$$

$$\underline{\underline{y = K \cdot e^{\arcsin x}}}$$

i) $1 + y^2 - y' \cdot x^{10} = 0$

$$-\frac{dy}{dx} \cdot x^{10} = -1 - y^2$$

$$\frac{dy}{dx} \cdot x^{10} = 1 + y^2$$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{x^{10}}$$

$$\operatorname{arctg}(1+y^2) = \frac{x^{-9}}{-9} + c$$

$$\operatorname{arctg} y = -\frac{1}{9x^9} + c$$

$$\underline{\underline{y = \operatorname{tg}\left(-\frac{1}{9x^9} + c\right)}}$$

h) $y' \cdot y^7 - \frac{1}{x} = 0$

$$\frac{dy}{dx} \cdot y^7 = \frac{1}{x}$$

$$\int y^7 \cdot dy = \int \frac{1}{x}\, dx$$

$$\frac{y^8}{8} = \ln|x| + c$$

$$y^8 = 8(\ln|x| + c)$$

$$\underline{\underline{y = \sqrt[8]{8(\ln|x| + c)}}}$$

j) $\cos^2 y - y' \cdot x^5 = 0$

$$-\frac{dy}{dx} \cdot x^5 = -\cos^2 y$$

$$\frac{dy}{dx} \cdot x^5 = \cos^2 y$$

$$\int \frac{dy}{\cos^2 y} = \int \frac{dx}{x^5}$$

$$\operatorname{tg} y = \frac{x^{-4}}{-4} + c$$

$$\underline{\underline{y = \operatorname{arctg}\left(\frac{x^{-4}}{-4} + c\right)}}$$

3. Metodou separace proměnných řešte diferenciální rovnice:

a) $y' = \cos x$

$$\frac{dy}{dx} = \cos x$$

$$\int dy = \int \cos x\, dx$$

$$\underline{\underline{y = \sin x + c}}$$

b) $y' = \sqrt{1-y^2}$

$$\frac{dy}{dx} = \sqrt{1-y^2}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int dx$$

$$\arcsin y = x + c$$

$$\underline{\underline{y = \sin(x+c);\ y = \pm 1}}$$

c) $x^2 y' + y^2 = 0$

$$x^2 \cdot \frac{dy}{dx} = -y^2$$

$$\int \frac{dy}{y^2} = \int -\frac{dx}{x^2}$$

$$-\frac{1}{y} = \frac{1}{x} + c \qquad / \cdot y \cdot x$$

$$-x = y + cxy$$

$$-x = y(1 + cx)$$

$$\underline{\underline{y = \frac{-x}{1+cx}\ ;\ y = 0}}$$

d) $y \cdot y' + x = 0$

$y \cdot \frac{dy}{dx} = -x$

$y\,dy = -x\,dx$

$\frac{y^2}{2} = -\frac{x^2}{2} + c \quad / \cdot 2$

$y^2 = -x^2 + 2c$

$\underline{y^2 = 2c - x^2}$

ve výsledcích? $y^2 = c^2 - x^2$?

e) $x \cdot y' = \sqrt{x} \cdot \cos^2 y$

$x \cdot \frac{dy}{dx} = \sqrt{x} \cdot \cos^2 y$

$\frac{dy}{\cos^2 y} = \frac{\sqrt{x}}{x} dx \qquad x^{\frac{1}{2}} \cdot x^{-1}$

$\int \frac{dy}{\cos^2 y} = \int x^{-\frac{1}{2}} \cdot dx$

$\operatorname{tg} y = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$\underline{\operatorname{tg} y = 2\sqrt{x} + c \;;\; y = \frac{\pi}{2} + k\pi}$

f) $2y'\sqrt{x} = y$

$2 \cdot \frac{dy}{dx} \cdot \sqrt{x} = y$

$\int \frac{dy}{y} = \int \frac{1}{2\sqrt{x}}\, dx$

$\ln|y| = \frac{1}{2} \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$\ln|y| = \sqrt{x} + c$

$|y| = e^{\sqrt{x} + c} \qquad \pm e^c = K$

$\underline{y = K \cdot e^{\sqrt{x}}}$

g) $y + y' \operatorname{cotg} x = 0$

$\frac{dy}{dx} \cdot \operatorname{cotg} x = -y$

$\frac{dy}{y} = \frac{-1}{\operatorname{cotg} x}\, dx$

$\int \frac{dy}{y} = -\int \operatorname{tg} x\, dx$

$\ln|y| = \ln|\cos x| + c \qquad c = \ln K, K > 0$

$\ln|y| = \ln|\cos x| + \ln K$

$\ln|y| = \ln|\cos x| \cdot K \qquad K = H, H \in R$

$\underline{y = H \cos x}$

h) $xy' = y - 1$

$x \cdot \frac{dy}{dx} = y - 1$

$\int \frac{dy}{y-1} = \int \frac{dx}{x}$

$\ln|y-1| = \ln|x| + c \qquad c = \ln K, K > 0$

$\ln|y-1| = \ln|x| + \ln K$

$\ln|y-1| = \ln|x| \cdot K \qquad K = H, H \in R$

$y - 1 = Hx$

$\underline{y = Hx + 1}$

i) $y - xy' = 3xy$

$-x \cdot \frac{dy}{dx} = 3xy - y$

$-x \cdot \frac{dy}{dx} = y(3x - 1)$

$-x\,dy = y(3x-1)\,dx$

$\frac{dy}{y} = \frac{3x - 1}{-x}\, dx$

$\int \frac{dy}{y} = \int \left(-3 + \frac{1}{x}\right) dx$

$\ln|y| = -3x + \ln|x| + c$

$|y| = e^{-3x + \ln|x| + c}$

$|y| = e^{-3x} \cdot e^{\ln|x|} \cdot e^c$

$|y| = e^{-3x} \cdot |x| \cdot \ln e \cdot e^c$

$\pm e^c = K$

$y = e^{-3x} \cdot x \cdot K$

$\underline{y = K \cdot x \cdot e^{-3x}}$

j) $2xy\cdot y' = y^2-3$

$2xy\cdot\frac{dy}{dx} = y^2-3$

$2xy\cdot dy = (y^2-3)\cdot dx$

$\int\frac{y}{y^2-3}dy = \int\frac{1}{2x}dx$

$\frac{1}{2}\int\frac{2y}{y^2-3}dy = \frac{1}{2}\int\frac{1}{x}dx$

$\ln|y^2-3| = \ln|x|+c \quad c=\ln K,\ K>0$

$\ln|y^2-3| = \ln|x|+\ln K$

$\ln|y^2-3| = \ln|x|\cdot K \quad K=H,\ H\in R$

$y^2-3 = Hx$

$y^2 = Hx+3$

l) $\frac{y'}{\cos x} - (5x+2)\cdot y^2 = 0$

$\frac{y'}{\cos x} = y^2(5x+2)$

$\frac{dy}{dx} = y^2\cdot(5x+2)\cdot\cos x$

$\int\frac{dy}{y^2} = \int(5x+2)\cdot\cos x\ dx$

$\left|\begin{matrix} u=5x+2 & v'=\cos x \\ u'=5 & v=\sin x\end{matrix}\right|$

$-\frac{1}{y} = (5x+2)\cdot\sin x - \int 5\sin x\ dx$

$-\frac{1}{y} = (5x+2)\cdot\sin x + 5\cos x + c$

$y = -\frac{1}{(5x+2)\cdot\sin x+5\cos x+c}\ ;\ y=0$

k) $y'\cdot e^{6-5x} = y^2$

$\frac{dy}{dx}\cdot e^{6-5x} = y^2$

$\frac{dy}{y^2} = \frac{1}{e^{6-5x}}dx$

$\int\frac{dy}{y^2} = \int e^{5x-6}dx \quad |\cdot 5$

$5\int\frac{dy}{y^2} = 5\int e^{5x-6}dx$

$-\frac{5}{y} = 5\cdot\frac{e^{5x-6}}{5}+c$

$-\frac{5}{y} = e^{5x-6}+c$

$y = \frac{-5}{e^{5x-6}+c}\ ;\ y=0$

4. Metodou variace konstanty najděte obecné řešení diferenciální rovnice:

a) $y' + 3y = \frac{x^2+5x+1}{e^{3x}}$

$$y' + 3y = 0$$

$$\frac{dy}{dx} = -3y$$

$$\int \frac{dy}{y} = \int -3\,dx$$

$$\ln|y| = -3x + c$$

$$|y| = e^{-3x+c}$$

$$|y| = e^{-3x} \cdot e^{c} \qquad \pm e^{c} = K$$

$$y = e^{-3x} \cdot K$$

$$y = e^{-3x} \cdot K(x)$$

$$y' = e^{-3x} \cdot (-3) \cdot K(x) + e^{-3x} \cdot K'(x)$$

$$\cancel{e^{-3x} \cdot (-3) \cdot K(x)} + e^{-3x} \cdot K'(x) + \cancel{3 \cdot e^{-3x} \cdot K(x)} = \frac{x^2+5x+1}{e^{3x}}$$

$$e^{-3x} \cdot K'(x) = (x^2+5x+1) \cdot e^{-3x}$$

$$K'(x) = x^2+5x+1$$

$$K(x) = \int (x^2+5x+1)\,dx =$$

$$= \frac{x^3}{3} + 5\frac{x^2}{2} + x + H$$

$$y = \left(\frac{1}{3}x^3 + \frac{5}{2}x^2 + x + H\right) \cdot e^{-3x}$$

b) $y' - y \cdot \operatorname{tg} x = \frac{x+1}{\cos x}$

$$y' - y \cdot \operatorname{tg} x = 0$$

$$\frac{dy}{dx} = y \operatorname{tg} x$$

$$\int \frac{dy}{y} = \int \operatorname{tg} x\,dx$$

$$\ln|y| = -\ln|\cos x| + c \qquad c = \ln k, k > 0$$

$$\ln|y| = \ln K - \ln|\cos x|$$

$$\ln|y| = \ln \frac{K}{|\cos x|} \qquad K = H, H \in R$$

$$y = \frac{H}{\cos x}$$

$$y = \frac{H(x)}{\cos x}$$

$$y' = \frac{H'(x) \cdot \cos x - H(x) \cdot (-\sin x)}{\cos^2 x}$$

$$\frac{H'(x) \cdot \cos x}{\cos^2 x} + \cancel{\frac{H(x) \cdot \sin x}{\cos^2 x}} - \frac{H(x)}{\cos x} \cdot \frac{\sin x}{\cos x} = \frac{x+1}{\cos x}$$

$$\frac{H'(x)}{\cos x} = \frac{x+1}{\cos x}$$

$$H'(x) = x+1$$

$$H(x) = \int (x+1)\,dx = \frac{x^2}{2} + x + L$$

$$y = \frac{1}{\cos x}\left(\frac{x^2}{2} + x + L\right)$$

c) $y' - 3x^2 y = 3x^2 \cdot e^{x^3}$

$y' - 3x^2 y = 0$

$\frac{dy}{dx} = 3x^2 y$

$\frac{dy}{y} = 3x^2 dx$

$\ln|y| = 3\frac{x^3}{3} + c$

$\ln|y| = x^3 + c$

$|y| = e^{x^3 + c}$

$|y| = e^{x^3} \cdot e^c \qquad \pm e^c = K$

$y = K \cdot e^{x^3}$

$y = K(x) \cdot e^{x^3}$

$y' = K'(x) \cdot e^{x^3} + K(x) \cdot e^{x^3} \cdot 3x^2$

$K'(x) \cdot e^{x^3} + \cancel{K(x) \cdot e^{x^3} \cdot 3x^2 - 3x^2 \cdot K(x) \cdot e^{x^3}} = 3x^2 \cdot e^{x^3}$

$K'(x) \cdot e^{x^3} = 3x^2 \cdot e^{x^3}$

$K'(x) = 3x^2$

$K(x) = \int 3x^2 dx = 3\frac{x^3}{3} + H = x^3 + H$

$y = (x^3 + H) \cdot e^{x^3}$

d) $-3y' + 30y = 8e^{9x}$

$-3y' + 30y = 0$

$-3 \cdot \frac{dy}{dx} = -30y \qquad /:(-3)$

$\frac{dy}{dx} = 10y$

$\int \frac{dy}{y} = \int 10\, dx$

$\ln|y| = 10x + c$

$|y| = e^{10x + c}$

$|y| = e^{10x} \cdot e^c \qquad \pm e^c = K$

$y = e^{10x} \cdot K$

$y = e^{10x} \cdot K(x)$

$y' = e^{10x} \cdot 10 \cdot K(x) + e^{10x} \cdot K'(x)$

$-3 \cdot [e^{10x} \cdot 10 \cdot K(x) + e^{10x} \cdot K'(x)] + 30e^{10x} \cdot K(x) = 8e^{9x}$

$-3e^{10x} \cdot K'(x) = 8e^{9x}$

$K'(x) = -\frac{8e^{9x}}{3e^{10x}}$

$K'(x) = -\frac{8}{3e^x}$

$K'(x) = -\frac{8}{3}e^{-x}$

$K(x) = \int -\frac{8}{3}e^{-x} dx = -\frac{8}{3} \cdot \frac{e^{-x}}{-1} + H =$

$= \frac{8}{3} \cdot e^{-x} + H$

$y = e^{10x} \cdot \left(\frac{8}{3}e^{-x} + H\right)$

$y = H \cdot e^{10x} + \frac{8}{3}e^{9x}$

e) $y' - y\cdot\cos x = e^{\sin x}$

$y' - y\cdot\cos x = 0$

$\frac{dy}{dx} = y\cdot\cos x$

$\int\frac{dy}{y} = \int\cos x\,dx$

$\ln|y| = \sin x + c$

$|y| = e^{\sin x + c}$

$|y| = e^{\sin x}\cdot e^{c} \qquad \pm e^{c} = k$

$y = k\cdot e^{\sin x}$

$y = k(x)\cdot e^{\sin x}$

$y' = k'(x)\cdot e^{\sin x} + k(x)\cdot e^{\sin x}\cdot\cos x$

$k'(x)\cdot e^{\sin x} + k(x)\cdot e^{\sin x}\cdot\cos x - k(x)\cdot e^{\sin x}\cdot\cos x = e^{\sin x}$

$k'(x)\cdot e^{\sin x} = e^{\sin x}$

$k'(x) = 1$

$k(x) = \int 1\,dx = x + H$

$y = (x+H)\cdot e^{\sin x}$

f) $xy' - 2y = x^2\ln x$

$xy' - 2y = 0$

$x\cdot\frac{dy}{dx} = 2y$

$\int\frac{dy}{y} = \int\frac{2}{x}dx$

$\ln|y| = 2\ln|x| + c \qquad c = \ln k,\ k>0$

$\ln|y| = \ln|x|^2 + \ln k$

$\ln|y| = \ln|x|^2\cdot k \qquad k = H,\ H\in\mathbb{R}$

$y = Hx^2$

$y = H(x)\cdot x^2$

$y' = H'(x)\cdot x^2 + H(x)\cdot 2x$

$x\cdot\left(H'(x)\cdot x^2 + H(x)\cdot 2x\right) - 2\cdot H(x)\cdot x^2 = x^2\ln x$

$H'(x)\cdot x^3 + H(x)\cdot 2x^2 - H(x)\cdot 2x^2 = x^2\ln x$

$H'(x)\cdot x^3 = x^2\ln x$

$H'(x)\cdot x = \ln x$

$H'(x) = \frac{\ln x}{x}$

$H(x) = \int\frac{\ln x}{x}dx = \left|\begin{matrix} t = \ln x \\ dt = \frac{1}{x}dx\end{matrix}\right|$

$H(x) = \int t\,dt = \frac{t^2}{2} + L =$

$= \frac{\ln^2 x}{2} + L$

$y = \left(\frac{\ln^2 x}{2} + L\right)\cdot x^2$

$y = \frac{1}{2}x^2\ln^2 x + Lx^2$

g) $y' - y \cdot \cot g x = (x+1) \cdot \sin x$

$$y' - y \cdot \cot g x = 0$$

$$\frac{dy}{dx} = y \cdot \cot g x$$

$$\int \frac{dy}{y} = \int \cot g x \, dx$$

$$\ln|y| = \ln|\sin x| + c \qquad c = \ln K, K>0$$

$$\ln|y| = \ln|\sin x| + \ln K$$

$$\ln|y| = \ln|\sin x| \cdot K \qquad K = H, H \in R$$

$$y = H \cdot \sin x$$

$$y = H(x) \cdot \sin x$$

$$y' = H'(x) \cdot \sin x + H(x) \cdot \cos x$$

$$H'(x) \cdot \sin x + H(x) \cdot \cos x - H(x) \cdot \sin x \cdot \frac{\cos x}{\sin x} = (x+1) \cdot \sin x$$

$$H'(x) \cdot \sin x = (x+1) \cdot \sin x$$

$$H'(x) = x+1$$

$$H(x) = \frac{x^2}{2} + x + L$$

$$y = \left(\frac{x^2}{2} + x + L\right) \cdot \sin x$$

h) $2y' - 16y = 9e^{10x}$

$$2y' - 16y = 0$$

$$y' - 8y = 0$$

$$\frac{dy}{dx} = 8y$$

$$\int \frac{dy}{y} = \int 8 \, dx$$

$$\ln|y| = 8x + c$$

$$|y| = e^{8x+c}$$

$$|y| = e^{8x} \cdot e^{c} \qquad \pm e^{c} = K$$

$$y = K \cdot e^{8x}$$

$$y = K(x) \cdot e^{8x}$$

$$y' = K'(x) \cdot e^{8x} + K(x) \cdot e^{8x} \cdot 8$$

$$2\left(K'(x) \cdot e^{8x} + K(x) \cdot e^{8x} \cdot 8\right) - 16 \cdot K(x) \cdot e^{8x} = 9e^{10x}$$

$$2 \cdot K'(x) \cdot e^{8x} = 9e^{10x}$$

$$K'(x) = \frac{9}{2} e^{2x}$$

$$K(x) = \int \frac{9}{2} e^{2x} dx = \frac{9}{2} \cdot \frac{e^{2x}}{2} + H$$

$$K(x) = \frac{9}{4} e^{2x} + H$$

$$y = \left(\frac{9}{4} e^{2x} + H\right) \cdot e^{8x}$$

$$y = H \cdot e^{8x} + \frac{9}{4} e^{10x}$$

i) $y'+xy=x$

$$y'+xy=0$$
$$\frac{dy}{dx}=-xy$$
$$\int\frac{dy}{y}=\int -x\,dx$$
$$\ln|y|=\frac{-x^2}{2}+c$$
$$|y|=e^{-\frac{x^2}{2}+c}$$
$$|y|=e^{-\frac{x^2}{2}}\cdot e^{c} \qquad \pm e^{c}=K$$
$$\underline{y=K\cdot e^{-\frac{x^2}{2}}}$$

$$y=K(x)\cdot e^{-\frac{x^2}{2}}$$
$$y'=K'(x)\cdot e^{-\frac{x^2}{2}}+K(x)\cdot e^{-\frac{x^2}{2}}\cdot\left(-\frac{1}{2}\cdot 2x\right)$$
$$\underline{y'=K'(x)\cdot e^{-\frac{x^2}{2}}+K(x)\cdot e^{-\frac{x^2}{2}}\cdot(-x)}$$

$$K'(x)\cdot e^{-\frac{x^2}{2}}+K(x)\cdot e^{-\frac{x^2}{2}}\cdot(-x)+x\cdot K(x)\cdot e^{-\frac{x^2}{2}}=x$$
$$K'(x)\cdot e^{-\frac{x^2}{2}}=x$$
$$\frac{K'(x)}{e^{\frac{x^2}{2}}}=x$$
$$K'(x)=x\cdot e^{\frac{x^2}{2}}$$
$$K(x)=\int x\cdot e^{\frac{x^2}{2}}\,dx=\left|\begin{array}{l} t=\frac{x^2}{2} \\ dt=\frac{2x}{2}dx=x\,dx\end{array}\right|$$
$$K(x)=\int e^{t}\cdot dt=e^{t}+H=\underline{e^{\frac{x^2}{2}}+H}$$

$$y=\left(e^{\frac{x^2}{2}}+H\right)\cdot e^{-\frac{x^2}{2}}$$
$$y=e^{0}+H\cdot e^{-\frac{x^2}{2}}$$
$$\underline{\underline{y=1+H\cdot e^{-\frac{x^2}{2}}}}$$

j) $y'+2y=e^{-x}$

$$y'+2y=0$$
$$\frac{dy}{dx}=-2y$$
$$\int\frac{dy}{y}=\int -2\,dx$$
$$\ln|y|=-2x+c$$
$$|y|=e^{-2x+c}$$
$$|y|=e^{-2x}\cdot e^{c} \qquad \pm e^{c}=K$$
$$\underline{y=e^{-2x}\cdot K}$$

$$y=K(x)\cdot e^{-2x}$$
$$\underline{y'=K'(x)\cdot e^{-2x}+K(x)\cdot e^{-2x}\cdot(-2)}$$

$$K'(x)\cdot e^{-2x}+K(x)\cdot e^{-2x}\cdot(-2)+2\cdot K(x)\cdot e^{-2x}=e^{-x}$$
$$K'(x)\cdot e^{-2x}=e^{-x}$$
$$K'(x)=\frac{e^{-x}}{e^{-2x}}$$
$$K'(x)=e^{-x-(-2x)}=e^{x}$$
$$K(x)=\int e^{x}dx=e^{x}+H$$

$$y=(e^{x}+H)\cdot e^{-2x}$$
$$\underline{\underline{y=H\cdot e^{-2x}+e^{-x}}}$$