3D prostor a vektory

1. Jsou dány vektory $\vec{a} = (0; 2; 4)$, $\vec{b} = (1; 3; 5)$ a $\vec{c} = (6; 1; 3)$. Vypočtěte $|\vec{a}|$; $|\vec{b}|$; $|\vec{c}|$; $\vec{a}x(\vec{b}x\vec{c})$; $(\vec{a}x\vec{b})x\vec{c}$; $(\vec{a} + \vec{b}) \cdot (\vec{c} - \vec{a})$; $(\vec{b} + \vec{c})x(\vec{a} - \vec{b})$; $(\vec{a} \cdot \vec{b})^2 + (\vec{c}x\vec{a})^2$.

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$|\vec{a}| = \sqrt{0^2+2^2+4^2} = \sqrt{4+16} = \underline{\underline{\sqrt{20}}}$$

$$|\vec{b}| = \sqrt{1+9+25} = \underline{\underline{\sqrt{35}}}$$

$$|\vec{c}| = \sqrt{36+1+9} = \underline{\underline{\sqrt{46}}}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (0,2,4) \times (4,27,-17) = \underline{\underline{(-142,16,-8)}}$$

3 5 1 3
1 3 6 1
4, 27, -17

2 4 0 2
27 -17 4 27
-142, 16, -8

$$(\vec{a} \times \vec{b}) \times \vec{c} = (-2,4,-2) \times (6,1,3) = \underline{\underline{(14,-6,-26)}}$$

2 4 0 2
3 5 1 3
(-2, 4, -2)

4 -2 -2 4
1 3 6 1
14, -6, -26

$$(\vec{a}+\vec{b}) \cdot (\vec{c}-\vec{a}) = (1;5,9) \cdot (6,-1,-1) = 1 \cdot 6 + 5 \cdot (-1) + 9 \cdot (-1) = \underline{\underline{-8}}$$

$$(\vec{b}+\vec{c}) \times (\vec{a}-\vec{b}) = (7,4,8) \times (-1,-1,-1) = \underline{\underline{(4,-1,-3)}}$$

4 8 7 4
-1 -1 -1 -1
4, -1, -3

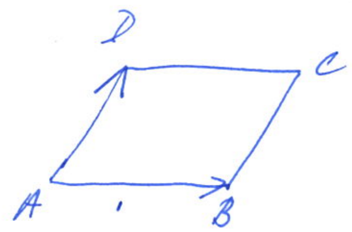
$$(\vec{a} \cdot \vec{b})^2 + (\vec{c} \times \vec{a})^2 = (0 \cdot 1 + 2 \cdot 3 + 4 \cdot 5)^2 + (-2,-24,12)^2 = 676 + (4,576,144) =$$

1 3 6 1
2 4 0 2
-2, -24, 12

$$= \underline{\underline{1400}}$$

?

2. Vypočítejte obsah rovnoběžníku, jehož vrcholy jsou body $A[0;0;0]$ a $B[1;2;3]$ a $D[3;2;1]$. Dopočítejte souřadnice bodu C.



$\vec{AD} = \vec{BC}$

$D - A = C - B$

$(3,2,1) = C - [1,2,3]$

$C = (3,2,1) + [1,2,3]$

$C = [4,4,4]$

$P = |\vec{AB} \times \vec{AD}| = |(-4,8,-4)| = \sqrt{16+64+16} = \sqrt{96} = \sqrt{2\cdot 48} = \sqrt{2\cdot 4\cdot 12} = \sqrt{2\cdot 4\cdot 3\cdot 4} =$

$= \sqrt{2}\cdot 2\cdot\sqrt{3}\cdot 2 = 4\sqrt{6}\ (j^2)$

$\vec{AB} = (1,2,3)$

$\vec{AD} = (3,2,1)$

$$\begin{array}{cccc} 2 & 3 & 1 & 2 \\ 2 & 1 & 3 & 2 \\ \hline \end{array}$$
$(-4, 8, -4)$

3. Ukažte, že body $A[2;1;0]$, $B[2;2;3]$, $C[0;1+\sqrt{40};0]$ tvoří vrcholy trojúhelníku. Pomocí vektorového součinu nalezněte jeho obsah.

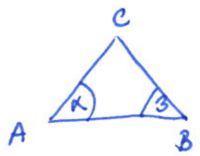
$\vec{AB} = B - A = (0,1,3)$

$\vec{AC} = C - A = (-2,\sqrt{40},0)$

$$\begin{array}{cccc} 1 & 3 & 0 & 1 \\ \sqrt{40} & 0 & -2 & \sqrt{40} \\ \hline \end{array}$$
$-3\sqrt{40}, -6, 2$

$P = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{9\cdot 40+36+4} = \frac{1}{2}\sqrt{400} = 10\ (j^2)$

4. Ukažte, že body $A[4;1;0]$, $B[4;-2;-3]$, $C[1;-5;-3]$ tvoří vrcholy trojúhelníku. Určete velikosti vnitřních úhlů a pomocí vektorového součinu nalezněte jeho obsah.



$\vec{AB} = (0,-3,-3)$

$\vec{AC} = (-3,-6,-3)$

$\cos\alpha = \frac{0+18+9}{\sqrt{18}\cdot\sqrt{54}} = \frac{27}{3\sqrt{2}\cdot 3\sqrt{6}} = \frac{3}{2\sqrt{3}}\ \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\alpha = 30° = \frac{\pi}{6}$

$\vec{BA} = (0,3,3)$

$\vec{BC} = (-3,-3,0)$

$\cos\beta = \frac{0-9+0}{\sqrt{18}\cdot\sqrt{18}} = \frac{-9}{3\sqrt{2}\cdot 3\sqrt{2}} = \frac{-1}{2}$

$\beta = 120° = \frac{2\pi}{3}$

$\vec{CA} = (3,6,3)$

$\vec{CB} = (3,3,0)$

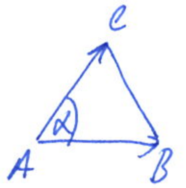
$\cos\gamma = \frac{9+18}{\sqrt{54}\cdot\sqrt{18}} = \frac{\sqrt{3}}{2}$

$\gamma = 30° = \frac{\pi}{6}$

$$\begin{array}{cccc} -3 & -3 & 0 & -3 \\ -6 & -3 & -3 & -6 \\ \hline \end{array}$$
$-9, 9, 9$

$P = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{81+81+81} = \frac{1}{2}\sqrt{243} = \frac{1}{2}\sqrt{3\cdot 81} = \frac{9\sqrt{3}}{2}\ (j^2)$

5. Ukažte, že body $A[2;-4;9]$, $B[-1;-4;5]$, $C[6;-4;6]$ tvoří vrcholy trojúhelníku. Určete velikost vnitřního úhlu α a pomocí vektorového součinu nalezněte jeho obsah.



$\vec{AB} = (-3, 0, -4)$

$\vec{AC} = (4, 0, -3)$

$$\cos\alpha = \frac{-12+0+12}{\sqrt{9+16}\cdot\sqrt{16+9}} = 0$$

$$\alpha = 90^\circ = \frac{\pi}{2}$$

$$P = \frac{1}{2}\sqrt{25^2} = \frac{1}{2}\cdot 25 = \frac{25}{2}\ (j^2)$$

$$\begin{matrix} 0 & -4 & -3 & 0 \\ 0 & -3 & 4 & 0 \end{matrix}$$

$0, -25, 0$

6. Jsou dány vektory $\vec{a} = (1;4;3)$, $\vec{b} = (3;0;2)$ a $\vec{c} = (6;0;4)$. Zjistěte, zda jsou lineárně závislé nebo nezávislé.

$$\alpha_1\cdot\vec{a} + \alpha_2\cdot\vec{b} + \alpha_3\cdot\vec{c} = \vec{0}$$

$$\alpha_1(1,4,3) + \alpha_2(3,0,2) + \alpha_3(6,0,4) = (0,0,0)$$

$\alpha_1 + 3\alpha_2 + 6\alpha_3 = 0$

$4\alpha_1 = 0 \quad \Rightarrow \alpha_1 = 0$

$3\alpha_1 + 2\alpha_2 + 4\alpha_3 = 0$

$3\alpha_2 + 6\alpha_3 = 0 \quad |:3$

$2\alpha_2 + 4\alpha_3 = 0 \quad |:(-2)$

$\alpha_2 + 2\alpha_3 = 0$

$-\alpha_2 - 2\alpha_3 = 0$

$0 = 0$

vektory jsou LZ

nebo: $\begin{vmatrix} 1 & 4 & 3 \\ 3 & 0 & 2 \\ 6 & 0 & 4 \end{vmatrix} = 0+48+0-0-48-0 = 0 \Rightarrow$ LZ

7. Jsou dány vektory $\vec{a} = (1;3;5)$, $\vec{b} = (3;-2;-1)$ a $\vec{c} = (4;1;3)$. Zjistěte, zda jsou lineárně závislé nebo nezávislé.

$$\alpha_1(1,3,5) + \alpha_2(3,-2,-1) + \alpha_3(4,1,3) = \vec{0}$$

$\alpha_1 + 3\alpha_2 + 4\alpha_3 = 0$

$3\alpha_1 - 2\alpha_2 + \alpha_3 = 0$

$5\alpha_1 - \alpha_2 + 3\alpha_3 = 0$

$$\begin{vmatrix} 1 & 3 & 5 \\ 3 & -2 & -1 \\ 4 & 1 & 3 \end{vmatrix} = -6-12+15+40-27+1 = 11 \neq 0 \Rightarrow LN$$

$$\begin{pmatrix} 1 & 3 & 4 \\ 3 & -2 & 1 \\ 5 & -1 & 3 \end{pmatrix} \begin{matrix} |\cdot(-3) & |\cdot(-5) \end{matrix} \sim \begin{pmatrix} 1 & 3 & 4 \\ 0 & -11 & -11 \\ 0 & -16 & -17 \end{pmatrix} |:(-1) \begin{pmatrix} 1 & 3 & 4 \\ 0 & 1 & 1 \\ 0 & -16 & -17 \end{pmatrix} |\cdot 16 \sim \begin{pmatrix} 1 & 3 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$\alpha_1 + 3\alpha_2 + 4\alpha_3 = 0 \quad \Rightarrow \alpha_1 = 0$

$\alpha_2 + \alpha_3 = 0 \quad \Rightarrow \alpha_2 = 0$

$-\alpha_3 = 0 \quad \Rightarrow \alpha_3 = 0$

vektory jsou LN

8. Jsou dány vektory $\vec{a} = (1; 3; 5)$, $\vec{b} = (-3; 2; -1)$ a $\vec{c} = (4; 1; 6)$. Zjistěte, zda jsou lineárně závislé nebo nezávislé.

$$\alpha_1 (1,3,5) + \alpha_2 (-3,2,-1) + \alpha_3 (4,1,6) = (0,0,0)$$

$$\alpha_1 - 3\alpha_2 + 4\alpha_3 = 0$$
$$3\alpha_1 + 2\alpha_2 + \alpha_3 = 0$$
$$5\alpha_1 - \alpha_2 + 6\alpha_3 = 0$$

$$\begin{vmatrix} 1 & 3 & 5 \\ -3 & 2 & -1 \\ 4 & 1 & 6 \end{vmatrix} = 12 - 12 - 15 - 40 + 54 + 1 = 0$$

$\Rightarrow$ LZ

$$\begin{pmatrix} 1 & -3 & 4 \\ 3 & 2 & 1 \\ 5 & -1 & 6 \end{pmatrix} \begin{matrix} \cdot(-3) \; \cdot(-5) \\ + \\ + \end{matrix} \sim \begin{pmatrix} 1 & -3 & 4 \\ 0 & 11 & -11 \\ 0 & 14 & -14 \end{pmatrix} \begin{matrix} \\ :11 \\ :(-14) \end{matrix} \sim \begin{pmatrix} 1 & -3 & 4 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{matrix} \\ \cdot 1 \\ + \end{matrix} \sim \begin{pmatrix} 1 & -3 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -3 & 4 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\alpha_1 - 3\alpha_2 + 4\alpha_3 = 0$$
$$\alpha_2 - \alpha_3 = 0$$
$$\underline{\alpha_2 = \alpha_3}$$
$$\alpha_1 - 3\alpha_2 + 4\alpha_2 = 0$$
$$\underline{\underline{\alpha_1 = -\alpha_2}}$$

vektory jsou LZ

9. Jsou dány vektory $\vec{a} = (1; 1; 1)$, $\vec{b} = (2; 1; 3)$ a $\vec{c} = (3; \omega; 4)$. Zjistěte, pro které ω jsou lineárně závislé.

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & w & 4 \end{vmatrix} = 0$$

$$4 + 9 + 2w - 3 - 8 - 3w = 0$$
$$2 - w = 0$$
$$\underline{\underline{w = 2}}$$

10. Napište vektor $\vec{a} = (-3; 5; -4)$, jako lineární kombinaci vektorů $\vec{b} = (1; -1; 2)$, $\vec{c} = (0; 2; 3)$ a $\vec{d} = (-2; 4; -3)$.

$$\vec{a} = c_1 \cdot \vec{b} + c_2 \cdot \vec{c} + c_3 \cdot \vec{d}$$

$$(-3, 5, -4) = c_1 (1, -1, 2) + c_2 (0, 2, 3) + c_3 (-2, 4, -3)$$

$$\begin{aligned} c_1 - 2c_3 &= -3 \\ -c_1 + 2c_2 + 4c_3 &= 5 \\ 2c_1 + 3c_2 - 3c_3 &= -4 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -3 \\ -1 & 2 & 4 & 5 \\ 2 & 3 & -3 & -4 \end{array}\right) \begin{array}{c} \cdot 1 \ \ \cdot(-2) \\ + \\ + \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & -3 \\ 0 & 2 & 2 & 2 \\ 0 & 3 & 1 & 2 \end{array}\right) : 2 \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 1 & 2 \end{array}\right) \begin{array}{c} \cdot(-3) \\ + \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -1 \end{array}\right)$$

$$\begin{aligned} c_1 - 2c_3 &= -3 &&\Rightarrow c_1 - 2 \cdot \tfrac{1}{2} = -3 && c_1 = -2 \\ c_2 + c_3 &= 1 &&\Rightarrow c_2 + \tfrac{1}{2} = 1 \Rightarrow && c_2 = \tfrac{1}{2} \\ -2c_3 &= -1 &&\Rightarrow c_3 = \tfrac{1}{2} \end{aligned}$$

$$\vec{a} = -2\vec{b} + \frac{1}{2}\vec{c} + \frac{1}{2}\vec{d}$$

11. Napište vektor $\vec{a} = (1; 2; 1)$, jako lineární kombinaci vektorů $\vec{b} = (1; 1; 1)$, $\vec{c} = (1; 1; 0)$ a $\vec{d} = (1; 0; 0)$.

$$\vec{a} = c_1 \cdot \vec{b} + c_2 \cdot \vec{c} + c_3 \cdot \vec{d}$$

$$(1, 2, 1) = c_1 (1, 1, 1) + c_2 (1, 1, 0) + c_3 (1, 0, 0)$$

$$\begin{aligned} c_1 + c_2 + c_3 &= 1 \\ c_1 + c_2 &= 2 \\ c_1 &= 1 \end{aligned} \qquad \begin{aligned} c_3 &= -1 \\ c_2 &= 1 \end{aligned}$$

$$\vec{a} = \vec{b} + \vec{c} - \vec{d}$$

12. Pro která τ je vektor $\vec{a} = (5; 3; \tau)$, lineární kombinaci vektorů $\vec{b} = (1; 1; 1)$, $\vec{c} = (2; 1; 3)$?

$\vec{a} = c_1 \cdot \vec{b} + c_2 \cdot \vec{c}$

$c_1 + 2c_2 = 5$

$c_1 + c_2 = 3 \quad / \cdot (-1)$

$c_2 = 2$

$c_1 + 4 = 5$

$c_1 = 1$

$\vec{a} = 1 \cdot \vec{b} + 2 \cdot \vec{c}$

$(5, 3, \tau) = (1, 1, 1) + 2(2, 1, 3)$

$(5, 3, \tau) = (1, 1, 1) + (4, 2, 6)$

$\tau = 7$

13. Vypočtěte vektorový součin vektorů $\vec{a} x \vec{b}$ a ukažte, že výsledný vektor je kolmý na oba vektory $\vec{a}$ a $\vec{b}$.

a) $\vec{a} = (6; 0; -2)$, $\vec{b} = (0; 8; 0)$ $\{(16; 0; 48)\}$

b) $\vec{a} = (1; 1; -1)$, $\vec{b} = (2; 4; 6)$ $\{(10; -8; 2)\}$

c) $\vec{a} = \vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{b} = -\vec{i} + 5\vec{k}$ $\{(15; -3; 3)\}$

d) $\vec{a} = \left(t; 1; \frac{1}{t}\right)$, $\vec{b} = (t^2; t^2; 1)$ $\{(1 - t; 0; t^3 - t^2)\}$

a)
$$\begin{array}{cccc} 0 & -2 & 6 & 0 \\ 8 & 0 & 0 & 8 \\ \hline \end{array}$$
$16, 0, 48$

$\vec{a} \times \vec{b} = (16, 0, 48)$

$\vec{a} \cdot (\vec{a} \times \vec{b}) = 6 \cdot 16 + 0 \cdot 0 + (-2) \cdot 48 = 0 \Rightarrow$

$\Rightarrow \vec{a} \perp (\vec{a} \times \vec{b})$

$\vec{b} \cdot (\vec{a} \times \vec{b}) = 0 + 0 + 0 = 0 \Rightarrow \vec{b} \perp (\vec{a} \times \vec{b})$

b)
$$\begin{array}{cccc} 1 & -1 & 1 & 1 \\ 4 & 6 & 2 & 4 \\ \hline \end{array}$$
$10, -8, 2$

$\vec{a} \times \vec{b} = (10, -8, 2) = \vec{u}$

$\vec{a} \cdot \vec{u} = 1 \cdot 10 - 1 \cdot 8 - 1 \cdot 2 = 0 \Rightarrow \vec{a} \perp \vec{u}$

$\vec{b} \cdot \vec{u} = 2 \cdot 10 - 4 \cdot 8 + 6 \cdot 2 = 0 \Rightarrow \vec{b} \perp \vec{u}$

c) $\vec{a} = (1, 3, -2)$

$\vec{b} = (-1, 0, 5)$

$$\begin{array}{cccc} 3 & -2 & 1 & 3 \\ 0 & 5 & -1 & 0 \\ \hline \end{array}$$
$15, -3, 3$

$\vec{a} \times \vec{b} = (15, -3, 3) = \vec{u}$

$\vec{a} \cdot \vec{u} = 15 - 9 - 6 = 0 \Rightarrow \vec{a} \perp \vec{u}$

$\vec{b} \cdot \vec{u} = -15 + 0 + 15 = 0 \Rightarrow \vec{b} \perp \vec{u}$

d)
$$\begin{array}{cccc} 1 & \frac{1}{t} & t & 1 \\ t^2 & 1 & t^2 & t^2 \\ \hline \end{array}$$
$(1-t;\ t-t,\ t^3-t^2)$

$\vec{a} \times \vec{b} = (1-t, 0, t^3-t^2) = \vec{u}$

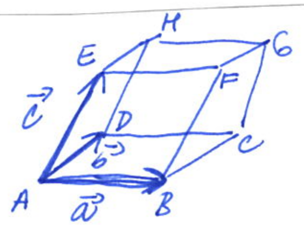
$\vec{a} \cdot \vec{u} = t(1-t) + 0 + t^2 - t = 0 \Rightarrow \vec{a} \perp \vec{u}$

$\vec{b} \cdot \vec{u} = t^2(1-t) + 0 + t^3 - t^2 = 0 \Rightarrow \vec{b} \perp \vec{u}$

14. Určete objem rovnoběžnostěnu ABCDEFGH, jsou-li dány body:

a) $A[1;0;1], B[3;-1;4], D[2;2;2], E[-1;3;5]$.

b) $A[2;-2;1], B[-1;1;3], D[3;2;2], F[-3;1;-2]$. .



$$V = |(\vec{a} \times \vec{b}) \cdot \vec{c}|$$

a) $\overrightarrow{AB} = (2,-1,3)$

$\overrightarrow{AD} = (1,2,1)$

$\overrightarrow{AE} = (-2,3,4)$

$$\begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ -2 & 3 & 4 \end{vmatrix} = 16+2+9+12+4-6 = 37$$

$V = 37\,(j^3)$

b) $\overrightarrow{AB} = (-3,3,2)$

$\overrightarrow{AD} = (1,4,1)$

$\overrightarrow{AE} = (-2,0,-5)$

$\overrightarrow{BF} = (-2,0,-5)$

$\overrightarrow{AE} = \overrightarrow{BF}$

$E - A = \overrightarrow{BF}$

$E - [2,-2,1] = (-2,0,-5)$

$E = (-2,0,-5) + [2,-2,1] = [0,-2,-4]$

$$\begin{vmatrix} -3 & 3 & 2 \\ 1 & 4 & 1 \\ -2 & 0 & -5 \end{vmatrix} = 60-6+0+16+15+0 = 85\,(j^3)$$

15. Jsou dány vektory $\vec{u} = (1;2;3)$, $\vec{v} = (1;1;1)$ a $\vec{w} = (1;3;1)$. Zjistěte, zda leží v jedné rovině?

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 1+2+9-3-2-3 = 4 \neq 0$$

$\Rightarrow$ vektory $\vec{u}, \vec{v}, \vec{w}$ neleží v jedné rovině

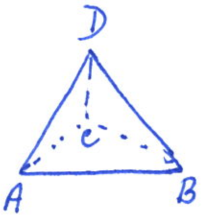
16. Je dán čtyřstěn ABCD, kde $A[-2;-1;-2]$, $B[1;4;0]$, $C[1;1;3]$, $D[2;5;3]$. Určete:

a) Obsah stěny BCD

b) Délku výšky v_D ve stěně BCD

c) Objem čtyřstěnu

d) Délku výšky čtyřstěnu kolmé na stěnu BCD



a) $P_{\Delta BCD} = \frac{1}{2}\left|\left(\overrightarrow{BC} \times \overrightarrow{BD}\right)\right| = \frac{1}{2}\sqrt{144+9+9} = \frac{1}{2}\sqrt{162} = \underline{\underline{\frac{9\sqrt{2}}{2}\ (j^2)}}$

$\overrightarrow{BC} = C - B = (0, -3, 3)$

$\overrightarrow{BD} = D - B = (1, 1, 3)$

$$\begin{array}{cccc} -3 & 3 & 0 & -3 \\ 1 & 3 & 1 & 1 \\ \hline \end{array}$$
$(-12, 3, 3)$

b) $|\overrightarrow{BC}| = \sqrt{0^2+3^2+3^2} = \sqrt{18} = 3\sqrt{2}$

$P_{\Delta BCD} = \frac{3\sqrt{2} \cdot v}{2}$

$\frac{9\sqrt{2}}{2} = \frac{3\sqrt{2}\, v}{2}$

$3 = v$

$\underline{\underline{v = 3\ (j)}}$

c) $V = \frac{1}{6}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right) \cdot \overrightarrow{AD}\right|$

$\overrightarrow{AB} = B - A = (3, 5, 2)$

$\overrightarrow{AC} = C - A = (3, 2, 5)$

$\overrightarrow{AD} = D - A = (4, 6, 5)$

$$\begin{vmatrix} 3 & 5 & 2 \\ 3 & 2 & 5 \\ 4 & 6 & 5 \end{vmatrix} = 30 + 100 + 36 - 16 - 75 - 90 = -15$$

$V = \frac{1}{6} \cdot |-15| = \frac{1}{6} \cdot 15 = \frac{15}{6} = \underline{\underline{\frac{5}{2}\ (j^3)}}$

d) $V = \frac{1}{3} \cdot P_{\Delta BCD} \cdot v$

$\frac{5}{2} = \frac{1}{3} \cdot \frac{9\sqrt{2}}{2} \cdot v \quad / \cdot 6$

$15 = 9\sqrt{2}\, v$

$v = \frac{15}{9\sqrt{2}} = \frac{5}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{3 \cdot 2} = \underline{\underline{\frac{5\sqrt{2}}{6}}}\ (j)$