POSLOUPNOST A JEJÍ LIMITA

① Vypočítejte

$$\lim_{n\to\infty} \frac{3^n+2^n}{3^n-2^n} \begin{matrix} \cdot\frac{1}{3^n} \\ \cdot\frac{1}{3^n} \end{matrix} = \lim_{n\to\infty} \frac{1+\left(\frac{2}{3}\right)^n}{1-\left(\frac{2}{3}\right)^n} = \frac{1+0}{1-0} = \underline{\underline{1}}$$

② Předpokládejme posloupnost, jejíž první členy jsou $\{3, 6, 12, 24, 48, \dots\}$. Vypočítejte součet prvních deseti členů této posloupnosti.

GP: $a_1 = 3 \quad q = 2$

$$s_n = a_1 \cdot \frac{1-q^n}{1-q}$$

$$s_{10} = 3 \cdot \frac{1-2^{10}}{1-2} = 3 \cdot \frac{1-1024}{-1} = 3 \cdot \frac{-1023}{-1} = \underline{\underline{3069}}$$

③ Vypočítejte

$$\lim_{n\to\infty} \frac{\sqrt[3]{8n^3+4n^2+2n-1}}{4n+5} \begin{matrix} \cdot\frac{1}{n} \\ \cdot\frac{1}{n} \end{matrix} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{8n^3+4n^2+2n-1}{n^3}}}{4+\frac{5}{n}} =$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{8+\frac{4}{n}+\frac{2}{n^2}-\frac{1}{n^3}}}{4+\frac{5}{n}} = \frac{\sqrt[3]{8+0+0-0}}{4+0} = \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$$

④ Vypočítejte, pro která $n \in N$ se hodnota výrazu $\frac{1}{n}$ liší od $\lim_{n\to\infty} \frac{1}{n}$ o méně než $\varepsilon = 0{,}1$.

$\lim_{n\to\infty} \frac{1}{n} = 0$

$$|a_n - A| < \varepsilon$$

$$\left|\frac{1}{n} - 0\right| < 0{,}1$$

$$\left|\frac{1}{n}\right| < \frac{1}{10}$$

$$\frac{1}{n} < \frac{1}{10}$$

$$10 < n$$

$$n > 10$$

$$\underline{\underline{n \geq 11}}$$

(5) Vypočítajte

$$\lim_{n\to\infty} \frac{(3n+2)^3}{(2n+3)(2n-3)} = \lim_{n\to\infty} \frac{27n^3+27n^2\cdot 2+9n\cdot 4+8}{4n^2-9} =$$

$$= \lim_{n\to\infty} \frac{27n^3+54n^2+36n+8}{4n^2-9} \begin{matrix} \cdot\frac{1}{n^2} \\ \cdot\frac{1}{n^2} \end{matrix} = \lim_{n\to\infty} \frac{27n+54+\frac{36}{n}+\frac{8}{n^2}}{4-\frac{9}{n^2}} =$$

$$= \lim_{n\to\infty} \frac{27n+54}{4} = \underline{\underline{\infty}}$$

(6) Vypočítajte

$$\lim_{n\to\infty} \frac{5-2n^2}{n^2+3n-2} \begin{matrix} \cdot\frac{1}{n^2} \\ \cdot\frac{1}{n^2} \end{matrix} = \lim_{n\to\infty} \frac{\frac{5}{n^2}-2}{1+\frac{3}{n}-\frac{2}{n^2}} = \frac{0-2}{1+0-0} = \underline{\underline{-2}}$$

(7) Vypočítajte

$$\lim_{n\to\infty} \frac{n!-(n+1)!}{n!} = \lim_{n\to\infty} \frac{n!-(n+1)n!}{n!} = \lim_{n\to\infty} \frac{n!\,(1-n-1)}{n!} =$$

$$= \lim_{n\to\infty} (-n) = \underline{\underline{-\infty}}$$

(8) Vypočítajte

$$\lim_{n\to\infty} \left(1+\frac{3}{2n-4}\right)^n = \left| \begin{matrix} \frac{3}{2n-4} = \frac{1}{m} \\ 3m = 2n-4 \\ 3m+4 = 2n \\ n = \frac{3m+4}{2} \\ n\to\infty \Rightarrow m\to\infty \end{matrix} \right| = \lim_{m\to\infty} \left(1+\frac{1}{m}\right)^{\frac{3}{2}m+2} =$$

$$= \lim_{m\to\infty} \left(1+\frac{1}{m}\right)^2 \cdot \left[\lim_{m\to\infty} \left(1+\frac{1}{m}\right)^m\right]^{\frac{3}{2}} = 1\cdot e^{\frac{3}{2}} = \underline{\underline{\sqrt{e^3}}} = \underline{\underline{e\sqrt{e}}}$$

9) Vypočítajte

$$\lim_{n\to\infty} \frac{5\cdot 2^{2n}+3^{2n}}{2^{2n}-2\cdot 3^{2n}} = \lim_{n\to\infty} \frac{5\cdot 4^{n}+9^{n}}{4^{n}-2\cdot 9^{n}} \quad \begin{matrix}\cdot \frac{1}{9^n} \\ \cdot \frac{1}{9^n}\end{matrix} =$$

$$= \lim_{n\to\infty} \frac{5\cdot\left(\frac{4}{9}\right)^{n}+1}{\left(\frac{4}{9}\right)^{n}-2} = \frac{5\cdot 0+1}{0-2} = -\frac{1}{2}$$

10) Vypočítajte

$$\lim_{n\to\infty} \left(\frac{n}{n-1}\right)^{3n+2} = \lim_{n\to\infty} \left(\frac{n-1+1}{n-1}\right)^{3n+2} = \lim_{n\to\infty} \left(1+\frac{1}{n-1}\right)^{3n+2} =$$

$$= \left| \begin{matrix} \frac{1}{n-1} = \frac{1}{m} \\ m = n-1 \\ n = m+1 \\ n\to\infty \Rightarrow m\to\infty \end{matrix} \right| = \lim_{m\to\infty} \left(1+\frac{1}{m}\right)^{3(m+1)+2} = \lim_{m\to\infty} \left(1+\frac{1}{m}\right)^{3m+5} =$$

$$= \lim_{m\to\infty} \left(1+\frac{1}{m}\right)^{5} \cdot \left[\lim_{m\to\infty} \left(1+\frac{1}{m}\right)^{m}\right]^{3} = 1\cdot e^{3} = e^{3}$$

11) Vypočítajte

$$\lim_{n\to\infty} \left(\frac{n^2+2}{n^2}\right)^{n^2-3} = \lim_{n\to\infty} \left(1+\frac{2}{n^2}\right)^{n^2-3} = \left| \begin{matrix} \frac{2}{n^2} = \frac{1}{m} \\ 2m = n^2 \\ n\to\infty \Rightarrow m\to\infty \end{matrix} \right| =$$

$$= \lim_{m\to\infty} \left(1+\frac{1}{m}\right)^{2m-3} = \lim_{m\to\infty} \left\{\left(1+\frac{1}{m}\right)^{-3} \cdot \left[\left(1+\frac{1}{m}\right)^{m}\right]^{2}\right\} = 1\cdot e^{2} = e^{2}$$

12) Vypočítajte

$$\lim_{n\to\infty} \frac{\sin n}{n} = \lim_{n\to\infty} \sin n \cdot \frac{1}{n}$$

a) $\forall n \in N$: $|\sin n| \leq 1 \Rightarrow$ OHRANIČENÁ

b) $\lim_{n\to\infty} \frac{1}{n} = 0$

$\Rightarrow \lim_{n\to\infty} \sin n \cdot \frac{1}{n} = 0$

(13) Vypočítejte, pro která $n \in N$ se hodnota výrazu $\frac{6}{2-3n}$ liší od hodnoty 0 o méně než $\varepsilon = 0{,}1$.

$$\left|\frac{6}{2-3n} - 0\right| < 0{,}1$$

$$\left|\frac{6}{-(3n-2)}\right| < \frac{1}{10}$$

$$\left|\frac{-6}{3n-2}\right| < \frac{1}{10}$$

$$\frac{6}{3n-2} < \frac{1}{10}$$

$$60 < 3n-2$$

$$62 < 3n$$

$$n > \frac{62}{3}$$

$$\underline{\underline{n \geq 21}}$$